2017 SYDNEY BOYS HIGH SCHOOL
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided with this paper
- Leave your answers in the simplest exact form, unless otherwise stated
- All necessary working should be shown in every question if full marks are to be awarded
- Marks may **NOT** be awarded for messy or badly arranged work
- In Questions 11–16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I Pages 2–6

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 8–19

90 marks

- Attempt Questions 11–16
- Allow about 2 hour and 45 minutes for this section

Examiner: *B.Kilmore*

Section I

10 marks
Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. A bag contains 10 green balls and m yellow balls. If a ball is drawn at random from the bag, the probability of drawing a yellow ball is $\frac{2}{3}$. The value of m is

(A) 5

(B) 10

(C) 15

(D) 20

2. If $f(x) = \log_e(2x)$, then $f'(1)$ is equal to

(A) $\frac{1}{2}$

(B) 1

(C) $\log_e 2$

(D) 2

3. A primitive of $e^{3x}+\frac{\sin(3x)}{3}$ is

(A) $\frac{e^{3x}}{3}-\frac{\cos(3x)}{9}$

(B) $e^{3x}-\cos(3x)$

(C) $3e^{3x}+3\cos(3x)$

(D) $e^{3x}+\cos(3x)$

4. The number of fish in an inlet is estimated using the equation $N=500-250\cos\left(\frac{\pi t}{6}\right)$ where t is the number of hours since low tide. Which statement is FALSE?

(A) The rate at which the number of fish is changing with respect to time is given by

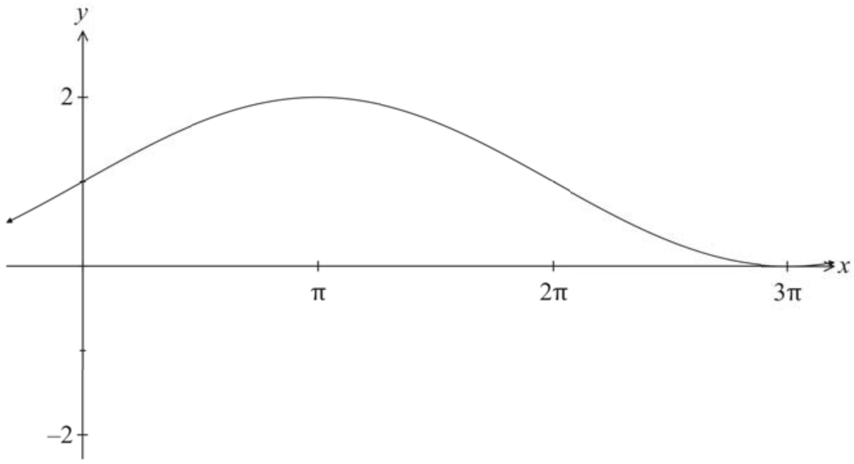
$$\frac{dN}{dt}=\frac{125\pi}{3}\sin\frac{\pi t}{6}$$

(B) The rate at which the number of fish is changing is increasing in the first 3 hours after low tide.

(C) The rate of change of the number of fish is positive in the first 6 hours.

(D) The maximum number of fish in the inlet at any time is 250.

5. The graph of $y = g(x)$ is shown. Let $f(x) = \sin x$.



Then $g(x)$ is given by

(A) $g(x) = f(2x) + 1$

(B) $g(x) = f(\frac{x}{2}) + 1$

(C) $g(x) = f(x) + 1$

(D) $g(x) = 2f(x) + 1$

6. The formula $V = 3x(9 - 4x^2)$ expresses the volume of an irregular solid as a function of one of its dimensions, x.
The volume of the solid is a maximum when the value of x is

(A) $\frac{3}{4}$

(B) $\frac{3}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{-\sqrt{3}}{2}$

7. If $y = \frac{3}{x^2} - 2e^{-x+4}$, then $\frac{dy}{dx}$ is equal to

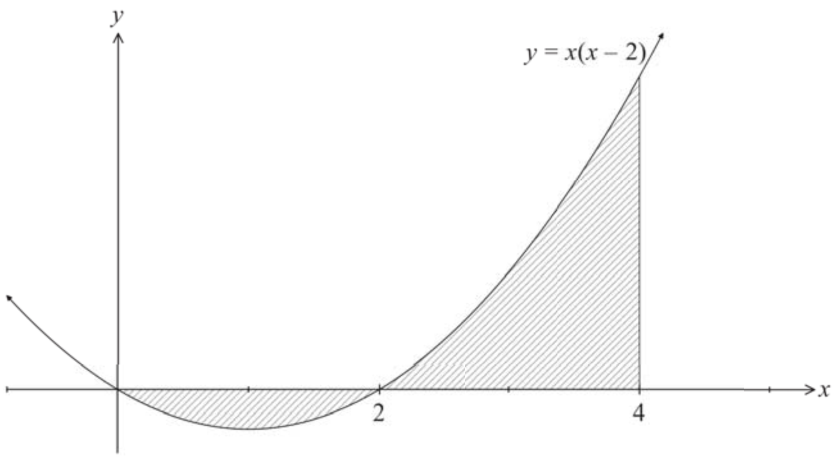
(A) $\frac{-6}{x} + 2e^{-x+4}$

(B) $\frac{-6}{x^3} - 2e^{-x+4}$

(C) $\frac{-6}{x} - 2e^{-x+4}$

(D) $\frac{-6}{x^3} + 2e^{-x+4}$

8. The shaded area bounded by the curve $y = x(x-2)$ and the x-axis, shown below, is equal to



(A) $5\frac{1}{3}$

(B) $\frac{52}{3}$

(C) $6\frac{2}{3}$

(D) 8

9. An approximate value of $\int_1^4 (x^2 + 1)\, dx$ is calculated using the trapezoidal rule using 4 function values. The value of this approximation is

(A) 49

(B) 24.5

(C) 20

(D) None of the above.

10. $\int_0^1 \frac{1}{3x+1}\, dx = \log_e k$ where k is equal to

(A) 1

(B) 2^2

(C) $2^{\frac{2}{3}}$

(D) $3^{\frac{1}{3}}$

Section II

90 marks
Attempt Questions 11–16

Allow about 2 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) USE A SEPARATE WRITING BOOKLET

a) If p = 0.42, evaluate, correct to 2 decimal places, **(1)**

$$\frac{1-p^2}{1+p^2}$$

b) Factorise completely: $54a^3+16$ **(2)**

c) Convert 24° to radians, giving your answer in terms of π **(1)**

d) Express $0.\dot{2}\dot{7}$ as a rational number in simplest form. **(1)**

e) One date is taken at random from those in May and another from those in June. **(2)**
What is the probability that both are the 13th of the month?
(Leave your answer as a fraction)

f) Solve $|2x+1|<x+3$ **(2)**

g) The nth term of an arithmetic sequence is given by $T_n=7+4n$. **(2)**
Find the last term which is less than 1000.

h) Differentiate with respect to x

(i) $\dfrac{3x}{7x^2 - 2}$ **(1)**

(ii) $e^{2x} \sin x$ **(2)**

(iii) $\tan(5x - 3)$ **(1)**

Question 12 (15 Marks) USE A SEPARATE WRITING BOOKLET

a) Solve for $0 \le \theta \le 2\pi$

(i) $\cos 2\theta = -\frac{1}{2}$ **(2)**

(ii) $\operatorname{cosec}\left(\theta + \frac{\pi}{4}\right) = \sqrt{2}$ **(2)**

b) Differentiate $y = (3x-1)^4(2x+5)^3$ **(2)**

c) Write down the domain of the function $f(x) = \ln(2x-3)$ **(1)**

d) Find the following integrals:

(i) $\int \frac{1}{\sqrt{x}}\, dx$ **(1)**

(ii) $\int \frac{dx}{2x}$ **(1)**

(iii) $\int \frac{dx}{\cos^2 x}$ **(1)**

(iv) $\int (5x+7)^6\, dx$ **(1)**

(v) $\int (1-e^{-x})^2\, dx$ **(2)**

e) Find the co-ordinates of the points of contact of the tangents to the curve $y = x^3 - 2x + 3$ which have an angle of inclination of 45° to the positive direction of the x-axis. **(2)**

Question 13 (15 Marks)   USE A SEPARATE WRITING BOOKLET

a) For what value of k will the equation $x^2-(k+2)x+(k-4)=0$ have one root that is the reciprocal of the other? **(2)**

b) Find $\int_{\frac{1}{6}}^{\frac{1}{2}} \cos(\pi x)\, dx$ **(2)**

c) Find the equation of the normal to $y=(\ln x)^2$ at the point where $x=e$. **(2)**

d) Find the area enclosed between the curves $y=\sin x$ and $y=\sqrt{3}\cos x$ for $0 \le x \le 2\pi$. **(3)**

e) A particle moves in a straight line such that its distance, x metres, from a fixed point O at any time t seconds is given by $x=5+\log_e(1+2t)$.

(i) Show that the particle will be 10 metres from O when $t=\dfrac{e^5-1}{2}$. **(2)**

(ii) Find the velocity and acceleration of the particle when it is 10 metres from O. **(2)**

(iii) Hence describe the position and velocity of the particle as $t \to \infty$. **(2)**

Question 14 (15 Marks) USE A SEPARATE WRITING BOOKLET

a) Ross decides to borrow \$150 000 to buy a luxury car.
For a 10-year loan, interest is compounded monthly on the balance still owing at a rate of 9% p.a.
The loan is to be repaid in equal monthly repayments of \$*M* at the end of each month.
Let A_n be the amount owing at the end of *n* months.

(i) Show that $A_2 = 150000(1.0075)^2 - M(1+1.0075)$ **(1)**

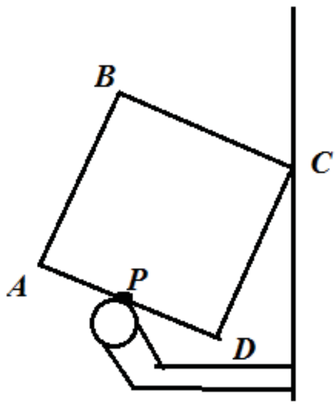
(ii) Write down an expression for A_n in terms of M . **(1)**

(iii) Calculate the value of M to 2 decimal places **(2)**

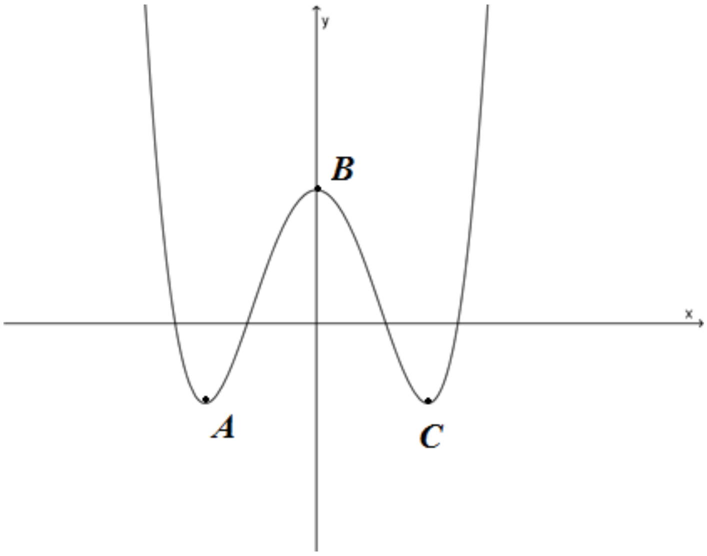
(iv) At the end of the 5th year, the interest rate changes to 8% p.a. compounding monthly. **(3)**
Assuming he does not change his repayments, how much sooner will he be able to pay off the loan?
(Answer to the nearest month)

b) A box of rectangular cross-section sits on a train luggage rack as shown with the point *C* touching the wall. **(2)**
P, the point in contact with the rack, is the midpoint of *AD*.
If *D* is 8 cm from the wall and *P*, the edge of the rack, is 20 cm from the wall, find how far *A* is from the wall.

Hint: Draw parallels to the wall through *A*, *P* and *D*.



(c) The graph below is of the function $y = f(x)$, where $f(x) = x^4 - 8x^2 + 10$.



(i) Find the coordinates of B. **(1)**

(ii) Find $f'(x)$. **(1)**

(iii) Show that the solutions of the equation $f'(x) = 0$ are $x = 0$, $x = -2$ and $x = 2$. **(2)**

(iv) Hence find the coordinates of A and C and confirm that they are minimum stationary points and that B is a maximum stationary point. **(2)**

Question 15 (15 Marks)   USE A SEPARATE WRITING BOOKLET

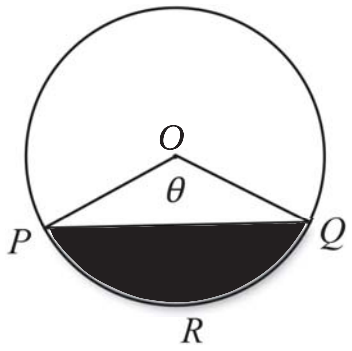
a) Find constants A, B and C such that $x^2 + x + 1 \equiv A(x+3)^2 + B(x+3) + C$. **(2)**

b) For the parabola $y^2 = -2(x-2)$, find

(i) The vertex, V, and the focus, S. **(1)**

(ii) The latus rectum is the chord through the focus that is perpendicular to the axis of symmetry of the parabola. Find the length of the latus rectum of this parabola. **(1)**

c) Find the area of the minor segment PRQ as shown in the diagram if the radius and the area of the sector $OPRQ$ are 12 cm and 48π cm^2 respectively. Let $\angle POQ = \theta$. **(3)**



d) A particle is moving in a horizontal straight line. At time t seconds it has displacement x metres to the right of a fixed point O on the line, given by $x = t(t-3)^2$, velocity, v m/s, and acceleration a m/s^2.

(i) Find expressions for v and a in terms of t. **(2)**

(ii) Find when the particle is moving towards O. **(1)**

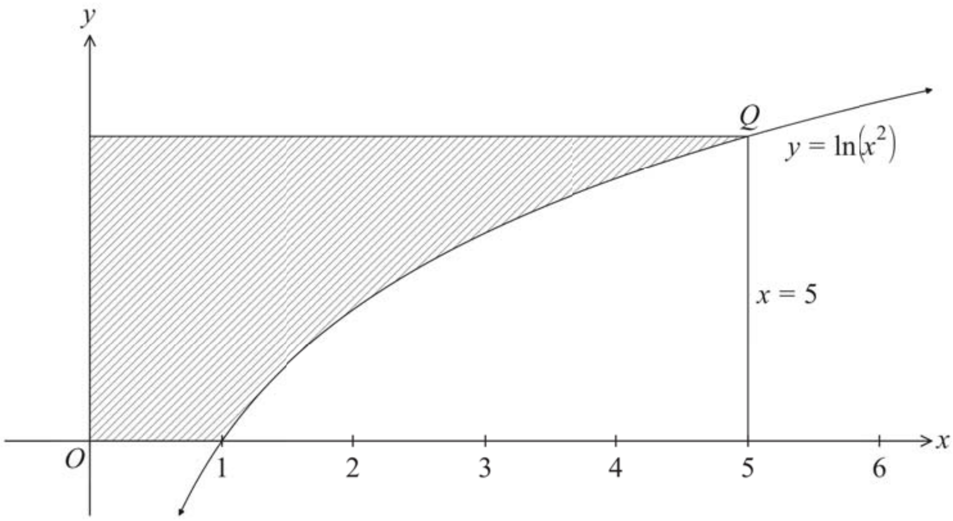
(iii) Find when the particle is moving towards O and slowing down. **(1)**

e) (i) Differentiate $x\log_e(x^2) - 2x$ and hence find $\int \log_e(x^2)\,dx$ **(2)**

(ii) The diagram below shows the curve $y = \log_e(x^2)$, $x > 0$, **(2)**

which meets the line $x = 5$ at Q.

Find the area of the shaded region.



Question 16 (15 Marks)   USE A SEPARATE WRITING BOOKLET

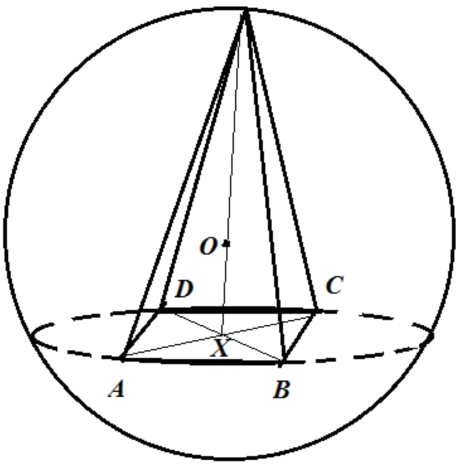
a) The probability that a biased coin lands heads up is p. I have a pair of identical biased coins. The coins show heads more often than normal coins show heads.

(i) Draw a tree diagram to show the possible outcomes when I toss the pair of biased coins together. **(1)**
Write the probabilities on each branch of the tree.

(ii) When I toss the coins, 25% of the time they land showing a head and a tail. Determine the probability that, on the next toss of the pair of coins, they will land with at least one of the coins showing a head. Give your answer correct to 3 decimal places. **(3)**

b) In the diagram the square *ABCD*, whose diagonals *AC* and *BD* meet at *X*, is the base of a right, square-based pyramid with apex *T* which is inscribed in a sphere of radius 1 metre with centre *O*. The vertices of the pyramid touch the inside of the sphere and $OX = x$ metres.

Note: The volume of a pyramid is $\frac{1}{3} \times \text{Area of the base} \times \text{Height}$.



(i) Show that the volume V m^3 of the inscribed pyramid is given by **(3)**

$$V = \frac{2}{3}(1 + x - x^2 - x^3)$$

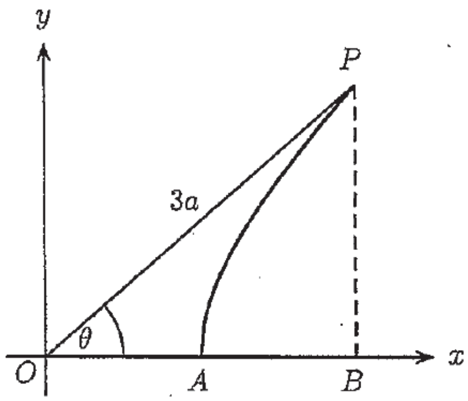
(ii) Hence find the maximum volume of the pyramid. **(3)**

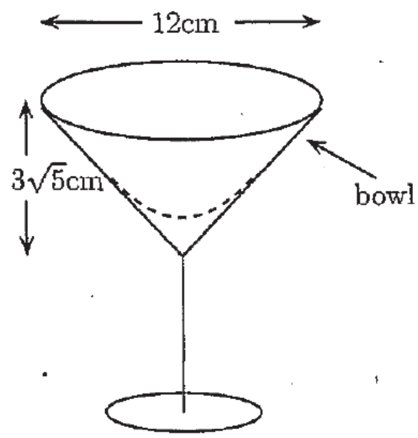
c) The diagram below on the left shows part of the graph of $y = \sqrt{x^2 - a^2}$ in the first quadrant intersects the x-axis at A.

P is the point on the curve such that $OP = 3a$ and $\angle POA = \theta$.

B is the foot of the perpendicular from P to the x-axis.

The region bounded by AB, PB and the curve is rotated about the x-axis to obtain the volume of liquid that can be placed in the conical bowl of a brass cup.





(i) Show that the volume of liquid that can be placed in the conical bowl is **(3)**

$$V = \frac{a^3\pi}{3}(27\cos^3\theta - 9\cos\theta + 2)$$

(ii) Given that the height OB of the conical bowl is $3\sqrt{5}$ cm and the diameter of its rim is 12 cm, find the volume of liquid that the cup will hold when it is full. **(2)**

End of paper

2017 SYDNEY BOYS HIGH SCHOOL
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Mathematics

Suggested Solutions

MC Answers

Q1	D
Q2	B
Q3	A
Q4	D
Q5	B
Q6	C
Q7	D
Q8	D
Q9	B
Q10	C

2U Y12 Assessment THSC 2017 Multiple choice solutions

Mean (out of 10): 8.47

1. $\frac{m}{10+m} = \frac{2}{3}$

$\therefore 3m = 20 + 2m$

$\therefore m = 20$ (D)

A	2
B	0
C	2
D	184

2. $f(x) = \log_e(2x)$

$\therefore f'(x) = \frac{1}{2x} \times 2$

$= \frac{1}{x}$

$\therefore f'(1) = 1$ (B)

A	7
B	168
C	9
D	4

3. $\int \left(e^{3x} + \frac{\sin(3x)}{3}\right)dx$

$= \frac{1}{3}e^{3x} + \frac{-\frac{1}{3}\cos 3x}{3} + c$

$= \frac{1}{3}e^{3x} - \frac{\cos 3x}{9} + c$ (A)

A	185
B	3
C	0
D	0

4. A $\frac{dN}{dt} = -250 \times -\sin\left(\frac{\pi t}{6}\right) \times \frac{\pi}{6}$

$= \frac{250\pi}{6} \sin \frac{\pi t}{6}$

$= \frac{125\pi}{3} \sin \frac{\pi t}{6}$ TRUE

B From A, $\frac{dN}{dt} > 0$ for $0 < t < 6$

TRUE

C From A, $\frac{dN}{dt} > 0$ for $0 < t < 6$

TRUE

D Max value occurs when

$\cos \frac{\pi t}{6} = -1$

$\therefore$ Max value is 750 FALSE

(D)

A	16
B	19
C	21
D	132

5. Graph of $g(x)$ has period of 4π

$\rightarrow f\left(\frac{x}{2}\right)$

$\therefore g(x) = f\left(\frac{x}{2}\right) + 1$ (B)

A	13
B	156
C	8
D	11

6. $V = 27x - 12x^3$

$V' = 27 - 36x^2$

$V'' = -72x$

For max volume

$27 - 36x^2 = 0$

$\therefore 4x^2 = 3$

$\therefore x = \pm\frac{\sqrt{3}}{2}$

Lengths are positive

$\therefore x = \frac{\sqrt{3}}{2}$

When $x = \frac{\sqrt{3}}{2}$, $V'' = -36\sqrt{3} < 0$

$\therefore$ Max volume when $x = \frac{\sqrt{3}}{2}$ (C)

A	7
B	12
C	165
D	4

7. $y' = 3 \times -2 \times x^{-3} - 2 \times e^{-x+4} \times -1$

$= \frac{-6}{x^3} + 2e^{-x+4}$ (D)

A	17
B	2
C	3
D	166

(8) Area $= -\int_0^2 (x^2 - 2x)\,dx + \int_2^4 (x^2 - 2x)\,dx$

$= -\left[\frac{x^3}{3} - x^2\right]_0^2 + \int_2^4 \left[\frac{x^3}{3} - x^2\right]$

$= -\left\{\left[\frac{8}{3} - 4\right] - [0]\right\} + \left\{\left[\frac{64}{3} - 16\right] - \left[\frac{8}{3} - 4\right]\right\}$

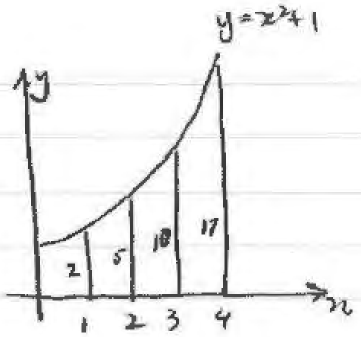
$= - -\frac{4}{3} + \frac{16}{3} - -\frac{4}{3}$

$= \frac{24}{3}$

$= 8$ (D)

A	30
B	1
C	11
D	146

9.



$\int_1^4 (x^2+1)\,dx$

$\approx \frac{1}{2}\{2 + 17 + 2(5 + 10)\}$

$= \frac{1}{2} \times 49$

$= 24.5$ (B)

A	7
B	149
C	2
D	30

10. $\int_0^1 \frac{1}{3x+1}\,dx$

$= \frac{1}{3}\left[\ln(3x+1)\right]_0^1$

$= \frac{1}{3}\{\ln 4 - \ln 1\}$

$= \ln 4^{1/3}$

$= \ln 2^{2/3}$

$\therefore k = 2^{2/3}$ (C)

A	3
B	15
C	150
D	20

2U TRIAL 2011 SOLUTIONS

QUESTION 11.

a) $p = .42$

$\frac{1-p^2}{1+p^2} = .70$

b) $54a^3 + 16$

$= 2(27a^3 + 8)$

$= 2((3a)^3 + 2^3)$

$= 2(3a+2)(9a^2 - 6a + 4)$

c) $\pi = 180°$

$\frac{\pi}{180} = 1°$

$\frac{24\pi}{180} = 24°$

$= \frac{2\pi}{15}$

d) $x = .2727\ldots$

$100x = 27.27\ldots$

$99x = 27$

$x = \frac{27}{99}$

$= \frac{3}{11}$

e) $\frac{1}{31} \times \frac{1}{30} = \frac{1}{930}$

f $|2x+1| < x+3$

$4x^2 + 4x + 1 < x + 3$

$3x^2 - 2x - 8 < 0$

$(3x+4)(x-2)$

g) $7 + 4n < 1000$

$4n < 993$

$n < 248.25$

$n = 248$

$T_{248} = 999$

No problems experienced by students on any of the previous.

h) $y = \frac{3x}{7x^2-2}$

$y' = \frac{(7x^2-2)3 - 3x(14x)}{(7x^2-2)^2}$

$= \frac{-21x^2 - 6}{(7x^2-2)^2}$

Many students thought there were common factors here and simplified wrongly.

i) $y = e^{2x} \sin x$

$y' = e^{2x}.\cos x + \sin x . 2e^{2x}$

$= e^{2x}(\cos x + 2\sin x)$

j) $y = \tan(5x-3)$

$y' = 5\sec^2(5x-3)$

Most students had no difficulties whatsoever with these questions.

QUESTION 12.

a (i) Solve $\cos 2\theta = -\frac{1}{2}$. for $0 \le \theta \le 2\pi$.

$$\therefore 0 \le 2\theta \le 4\pi$$

$$2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$$

$$\therefore \boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}.}$$

COMMENT The question was framed in radians and the answers should have been in radians also.

A significant number only provided the first two answers, clearly not considering $0 \le 2\theta \le 4\pi$.

(ii) $\operatorname{cosec}\left(\theta + \frac{\pi}{4}\right) = \sqrt{2}$. for $0 \le \theta \le 2\pi$.

$$\therefore \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \text{for } \frac{\pi}{4} \le \theta + \frac{\pi}{4} \le \frac{9\pi}{4}$$

$$\theta + \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}$$

$$\therefore \boxed{\theta = 0, \frac{\pi}{2}, 2\pi}$$

COMMENT. Wrong answers were caused by not considering $\frac{\pi}{4} \le \theta + \frac{\pi}{4} \le \frac{9\pi}{4}$.

b $y = (3x-1)^4(2x+5)^3$

$$\frac{dy}{dx} = 4\times 3(3x-1)^3(2x+5)^3 + 3\times 2(2x+5)^2(3x-1)^4$$

$$= 12(3x-1)^3(2x+5)^3 + 6(2x+5)^2(3x-1)^4$$

$$= 6(3x-1)^3(2x+5)^2\left[2(2x+5) + (3x-1)\right]$$

$$\boxed{= 6(3x-1)^3(2x+5)^2(7x+9)}$$

COMMENT In the general instructions it is clearly stated. "Leave your answers in simplest exact form, unless otherwise stated."

c Given $f(x) = \ln(2x-3)$

The domain is $2x-3>0$

$2x>3$

$\boxed{x>\frac{3}{2}}$

COMMENT. Well done. Some students had $x \geq \frac{3}{2}$. clearly $\ln 0$ is undefined.

d (i) $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\,dx$

$$= \frac{x^{\frac{1}{2}}}{1/2} + c$$

$$= \boxed{2\sqrt{x} + c}$$

COMMENT Well done.

(ii) $\int \frac{dx}{2x} = \frac{1}{2}\int \frac{dx}{x}$

$= \boxed{\frac{1}{2}\ln x + c}$

COMMENT $\frac{1}{2}\ln 2x + c_1$ was an acceptable answer. (c_1 is a different constant to c).

(iii) $\int \frac{dx}{\cos^2 x} = \int \sec^2 x \, dx$

$= \boxed{\tan x + c}$

COMMENT: simple question.

(iv) $\int (5x+7)^6 \, dx = \frac{(5x+7)^7}{5\times 7} + c.$

$= \boxed{\frac{(5x+7)^7}{35} + c}$

COMMENT: well done.

(v). $\int (1-e^{-x})^2 \, dx = \int (1 - 2e^{-x} + e^{-2x}) \, dx.$

$= \boxed{x + 2e^{-x} - \frac{1}{2}e^{-2x} + c}$

COMMENT Well done.

(e) $y = x^3 - 2x + 3$

$y' = 3x^2 - 2.$

Let $3x^2 - 2 = 1$ (since $\tan 45° = 1$)

$\therefore x = \pm 1$

$\therefore$ Points are $\boxed{(1,2) \text{ and } (-1,4)}$

COMMENT

straight forward. well done.

Question (13)

(a) $x^2-(k+2)x+(k-4)=0$

Let the roots be $\alpha, \frac{1}{\alpha}$

$\therefore (k-4)=1$ ✓

$k=5$ (2)

(b) $\int_{1/6}^{1/2} \cos(\pi x)\,dx$

$= \left[\frac{\sin \pi x}{\pi}\right]_{1/6}^{1/2}$ ✓

$= \frac{1}{\pi}-\frac{1}{2\pi} = \frac{1}{2\pi}$

[redacted] (2)

(c) $y=(\ln x)^2$

$\frac{dy}{dx} = \frac{2\ln x}{x}$ $(e,1)$

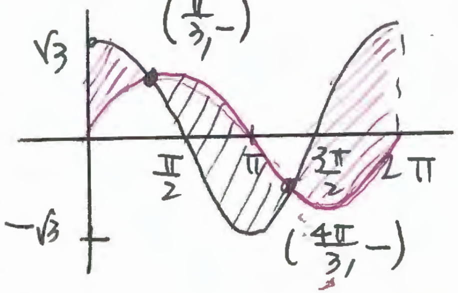
$\left.\frac{dy}{dx}\right|_{x=e} = \frac{2}{e}$, $m' = -\frac{e}{2}$ ✓

$y-1 = -\frac{e}{2}(x-e)$

$2y-2 = -ex+e^2$

$\therefore ex+2y = (2+e^2)$ (2)

(d) (3)



(i) $\frac{\sin x}{\cos x} = \sqrt{3}$

$x = \pi/3, \frac{4\pi}{3}$ ✓

(ii) $\int_{\pi/3}^{4\pi/3} (\sin x - \sqrt{3}\cos x)\,dx$

$= \left[-(\cos x + \sqrt{3}\sin x)\right]_{\pi/3}^{4\pi/3}$ ✓

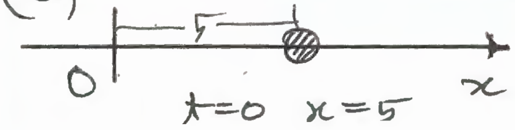
$= \left[-\left(-\frac{1}{2}-\frac{3}{2}\right)+\left(\frac{1}{2}+\frac{3}{2}\right)\right]$

$= 4$ ✓ $+\int_0^{\pi/3}(\sqrt{3}\cos x - \sin x)\,dx$

$+\int_{4\pi/3}^{2\pi}(\sqrt{3}\cos x - \sin x)\,dx$

$=(3)$ $=8$

(e)



$x = 5+\ln(1+2t)$

(i) When $t = \frac{e^5-1}{2}$ ✓

$x = 5+\ln(1+e^5-1)$

$= 5+\ln e^5$ ✓

$= 10$ (2)

(ii) $\frac{dx}{dt} = \frac{2}{1+2t}$ (2)

$\frac{d^2x}{dt^2} = \frac{-4}{(1+2t)^2}$

When $t = \frac{e^5-1}{2}$

$\frac{dx}{dt} = \boxed{2e^{-5}}$ ✓ 0.0135

$\ddot{x} = \boxed{-4e^{-10}}$ ✓ -0.000182

(iii) $x \to \infty$ and $v \to 0$
✓ ✓

(2)

Question 14

(i) $A_1 = 150000(1.0075) - M$

$A_2 = A_1(1.0075)A_1 - M$ ✓

(1) $= [150000(1.075) - M](1.0075)$

$= 150000(1.0075)^2 - M(1 + 1.0075)$

(ii) $A_n = 15000(1.0075)^n - M(1 + 1.0075 + \dots + 1.0075^{n-1})$ (1)

OR

$A_n = 150k(1.0075)^n - M\dfrac{[(1.0075)^n - 1]}{0.0075}$

(iii) $M = \dfrac{0.0075 \times [150000(1.0075)^{120}]}{(1.0075)^{120} - 1}$ ✓

$= 1900.14$ ✓ (2)

(iv) $A_{60} = 150k(1.0075)^{60} - \dfrac{(1900.14)[(1.0075)^{60} - 1]}{0.0075}$

$= 91535.73$ ✓

$A_n = 91535.73(1.0067)^n - \dfrac{(1900.14)[(1.0067)^n - 1]}{0.0067}$

$A_n = 0$

$\dfrac{(1900.14)[(1.0067)^n - 1]}{0.0067} = 91535.73(1.0067)^n$ ✓

$(1.0067)^n - 1 = 0.32276(1.0067)^n$

$0.67724(1.0067)^n = 1$

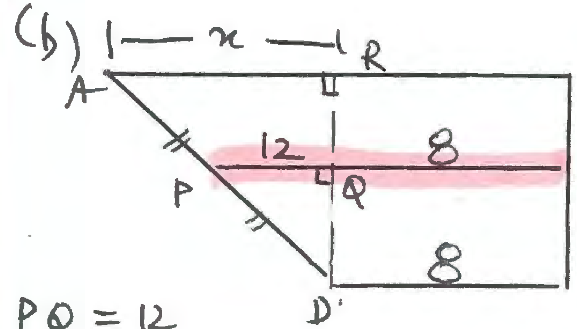
$(1.0067)^n = 1.476582$

$n = \dfrac{\log 1.476582}{\log 1.0067}$ (3)

$= 58.36 \doteqdot 59$ mth

∴ 1 month sooner ✓

(b)



$PQ = 12$

$\triangle PDQ \,|||\, \triangle ADR$

$\dfrac{AR}{12} = 2 \Rightarrow AR = 24$

$\therefore A = 24 + 8 = 32$ from wall (2)

(c)(i) $x = 0,\ y = 10$ (1)

$\therefore B(0, 10)$ ✓

(ii) $f'(x) = 4x(x^2 - 4)$ (1)

OR $4x^3 - 16x$ ✓

(iii) $f'(x) = 4x(x+2)(x-2)$

$= 0$

$\therefore x = 0, \pm 2.$ ✓ (2)

(iv) $x = \pm 2,\ y = -6.$

$\therefore A(-2, -6) \quad C(2, -6)$

$f''(x) = 12x^2 - 16$ (2)

$f''(-12) = 32 > 0 \ \therefore A, C$ min

$f''(0) = -16 < 0 \ \therefore B$ max

Question 15 Solutions

a) $RHS = A(x^2+6x+9)+B(x+3)+C$

$= Ax^2+6Ax+9A+Bx+3B+C$

$= Ax^2+(6A+B)x+9A+3B+C$

Since $x^2+x+1 \equiv Ax^2+(6A+B)x+9A+3B+C$

Equate coefficients

$\therefore A=1$

$6A+B=1$

$6+B=1$

$B=-5$

$9A+3B+C=1$

$9+3(-5)+C=1$

$9-15+C=1$

$-6+C=1$

$C=7$

$\therefore A=1,\ B=-5,\ C=7$ (1) mark

(1) mark for correct working out.

Marker's comments

- Common error was silly mistakes.

b) i) $y^2=-4a(x-h)$ (standard form)

$4a = 2$

$a = \frac{1}{2}$

$h = 2$

$\therefore$ Vertex $= (2,0)$ (0.5) mark

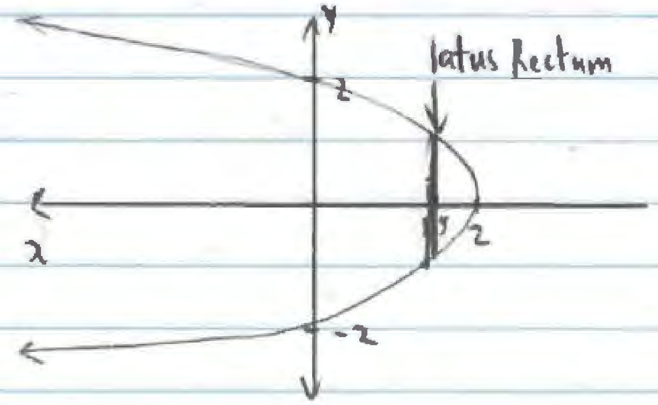
focus $= (1.5, 0)$ (0.5) mark

Marker's Comments

Most candidates were able to find the vertex correctly.

However, quite a number of candidates struggled to find the focus. Candidates whom drew a quick sketch of the graph were more successful in finding the focus.

(ii)



At $x=1.5$

$y^2=-2(1.5-2)$

$=1$

$\therefore y=\pm 1$

$\therefore$ length $= 2$ units.

(1) mark

Marker's Comments

- Candidates whom used the formula to find the length of the latus rectum correctly (4a) were given full marks.
- No half marks were given in this question.

1) Area of segment = Area of Sector OPRQ − Area of $\triangle OPQ$

Area of sector = $\frac{\theta}{2} \times r^2$

$48\pi = \frac{\theta}{2} \times 12^2$

$48\pi = \theta \times 72$

$\theta = \frac{2\pi}{3}$ (1 mark)

$\therefore$ Area of segment = $48\pi - \frac{1}{2} \times 12^2 \times \sin\frac{2\pi}{3}$ (1 mark)

$= 48\pi - 144 \times 0.5 \times \frac{\sqrt{3}}{2}$

$= 48\pi - 36\sqrt{3}$. cm^2 (1 mark.)

Marker's comments

Majority of all candidates were able to find ϴ correctly.

Many candidates did not read the GENERAL INSTRUCTIONS on the front page of the question booklet, where it states, "Leave your answer in the simplest EXACT form, unless otherwise stated".

d) i) $x = t(t^2 - 6t + 9)$

$= t^3 - 6t^2 + 9t$

$v = \dot{x} = 3t^2 - 12t + 9$ (1 mark)

$a = \ddot{x} = 6t - 12$ (1 mark.)

Marker's comments

Candidates whom expanded out the displacement function and differentiated to find the velocity and acceleration were more successful than candidates using the product rule to find the derivative.

(ii) $v < 0$

$3t^2 - 12t + 9 \quad 0$

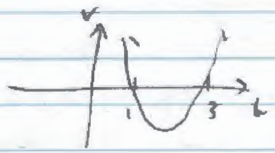
$3(t^2 - 4t + 3) \quad 0$

$t^2 - 4t + 3 \quad 0$

$(t-3)(t-1) \quad 0$

$\therefore 1 < t < 3$ seconds

(1 mark)



Marker's Comments

Most candidates were successful in this question.

(iii) Slowing down if $v < 0$ then $a > 0$

$6t - 12 > 0$

$6t > 12$

$t > 2$

$\therefore 2 < t < 3$ is when the particle is moving towards 0 and slowing down. (1 mark)

Marker's Comments

Poorly done by many candidates, as candidates need to follow the rule that if acceleration is in the opposite direction to velocity than the object will be slowing down. In part ii), since velocity is negative than we must have acceleration to be positive.

No half marks were given if candidates took acceleration as negative.

e) (i) let $y = x\log_e(x^2) - 2x$

$$\frac{dy}{dx} = \log_e(x^2) + x \times \frac{2x}{x^2} - 2$$ ① mark.

$$= \log_e(x^2) + 2 - 2$$

$$= \log_e(x^2)$$

$$\therefore \int \log_e(x^2)\,dx = x\log_e(x^2) - 2x + C$$ ① mark.

Marker's comments

Quite a number of candidates were unable to differentiate $x\log_e(x^2)$ successfully.

Candidates need to not forget the constant C when integrating.

(ii) Area = Area of rectangle $- \int_1^5 \ln(x^2)\,dx$

$$= \ln(25) \times 5 - \int_1^5 \ln(x^2)\,dx$$ ① mark.

$$= 5\ln(25) - \left[x\log_e(x^2) - 2x\right]_1^5$$

$$= 5\ln(25) - \left[(5\ln 25 - 10) - (1\ln 1 - 2)\right]$$

$$= 5\ln(25) - 5\ln 25 + 10 + 0 - 2$$

$$= 8 \text{ units}^2$$ ① mark.

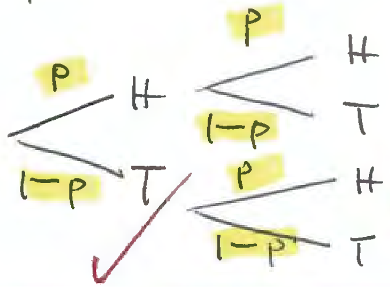
Marker's Comments

Candidates were able to successfully get the correct answer if they got part i) correct.

Question (16)

(a)

(i)



(1)

(ii) $2p - 2p^2 = \frac{1}{4}$

$8p^2 - 8p + 1 = 0$

$p = \frac{8 \pm 4\sqrt{2}}{16}$

$\left(\frac{2+\sqrt{2}}{4}\right)$ $\left(\frac{2-\sqrt{2}}{4}\right)$ ×

0.8535 ✓ 0.14645

(3)

P(At 1 head)

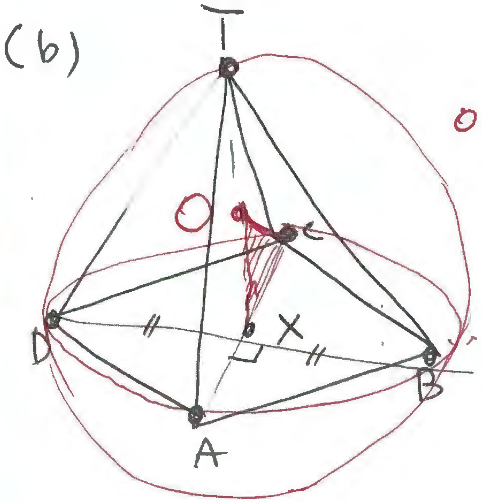
= 1 − P(TT).

$p = 0.8535$
$t = 0.1465$
$1 - t^2 = 0.979$
(1)

Note:

$p > \frac{1}{2}$ as heads is more likely

(b)



OC = 1

Area of ABCD.

$= \frac{1}{2} AC \times BC$

$= \frac{1}{2}(AC)^2$

$= \frac{1}{2}(2XC)^2 = 2XC^2$

(Square is a rhombus with equal diagonals bisecting each other at 90°)

$XC^2 = OC^2 - OX^2$

$= 1 - x^2$ ✓

Area of ABCD

$= 2(1 - x^2)$ ✓

$TX = OT + OX$

$= 1 + x$

∴ Volume of Pyramid

(3) $V = \frac{1}{3} \times 2(1-x^2)(1+x)$ ✓

$V = \frac{2}{3}(1 + x - x^2 - x^3)$

$= \frac{2}{3}(1 - 2x - 3x^2)$

$\frac{dV}{dx} = -\frac{2}{3}(3x-1)(x+1)$ ✓

Since $0 < x < 1$

$\frac{dV}{dx} = 0 \Rightarrow x = \frac{1}{3}$ ✓

and $\frac{d^2V}{dx^2} < 0$

$\frac{d^2V}{dx^2} = -\frac{4}{3}(1+3x)$

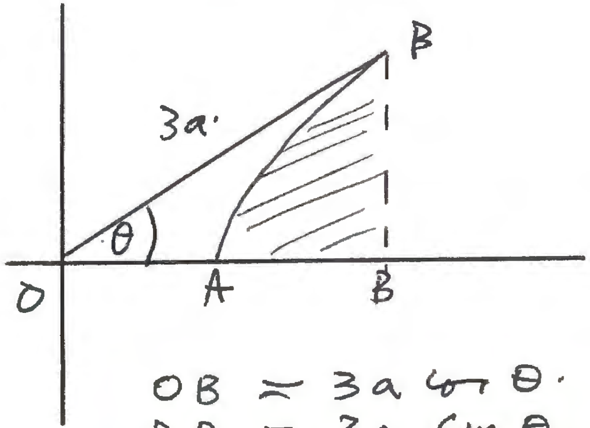
∴ $V_{max} = \frac{64}{81} m^3$ ✓

(3)

0.79

Question (16) (c)

(i)



$OB = 3a\cos\theta$.
$PB = 3a\sin\theta$.

$$V = \pi\int_a^{3a\cos\theta}(x^2 - a^2)\,dx$$

$$= \pi\left[\frac{x^3}{3} - a^2x\right]_a^{3a\cos\theta}$$

$$= \frac{a^3\pi}{3}\left[\frac{27a^3\cos^3\theta}{3} - 3a^3\cos\theta - \frac{a^3}{3} + a^3\right]$$

$$= \frac{a^3\pi}{3}(27\cos^3\theta - 9\cos\theta + 2)$$

(3)

$$\frac{3\sqrt{5}}{3a} = \cos\theta \qquad \frac{6}{3a} = \sin\theta$$

$\therefore\ a\cos\theta = \sqrt{5}, \quad a\sin\theta = 2$

$a^2 = 5 + 4 = 9 \Rightarrow a = 3.$

$\therefore\ \cos\theta = \frac{\sqrt{5}}{3}, \quad \sin\theta = \frac{2}{3}$

$\cos^3\theta = \frac{5\sqrt{5}}{27}, \quad 9\cos\theta = 3\sqrt{5}$

$\therefore\ V = 9\pi(5\sqrt{5} - 3\sqrt{5} + 2)$

$= 18\pi(\sqrt{5} + 1)$

$\doteq 183$

(2)